# Conformal Flattening of Brain Ventricles


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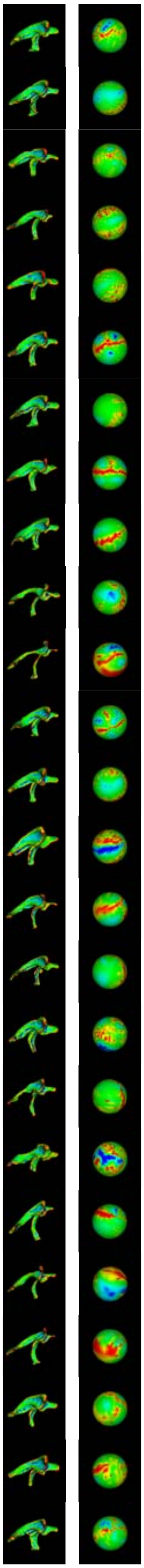
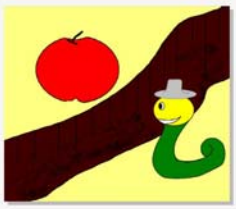
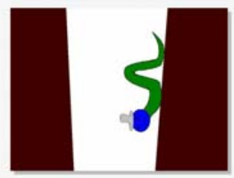


## Motivation



Hah! It's close!

## Motivation (cont.)



Not that close…

Since I can only walk on Surface…

## Algorithm (cont.)

1. Continuous formula:

$$z:\Sigma \to \mathbf{R}^2$$

$$\Delta z = \left(\frac{\partial}{\partial u} - i\frac{\partial}{\partial v}\right)\delta_p$$

2. Finite element method

$$D(z) := \frac{1}{2}\iint_{\Sigma}[|\nabla z|^2 + 2z(\frac{\partial}{\partial u} - i\frac{\partial}{\partial v})\delta_p]\mathrm{d}S$$

## Algorithm (cont.)

3. Discrete on mesh
   - Approximate *z* by Piecewise Linear function defined on each node.
   - So: $z = \mathbf{z}^r_P + i\mathbf{z}^i_P$

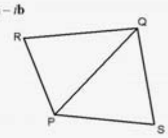
$$P = 1, \ldots, \textit{Number of triangles}$$

## Algorithm (cont.)

4. Conjugate gradient solver

$$\mathbf{D}(\mathbf{z}^r + i\mathbf{z}^i) = \mathbf{a} - i\mathbf{b}$$

$$\begin{cases} \mathbf{D}_{P,Q} = -\frac{1}{2}(ctg\angle R + ctg\angle S)\ (P \neq Q) \\ \mathbf{D}_{P,P} = -\sum_{P \neq Q}\mathbf{D}_{P,Q} \end{cases}$$

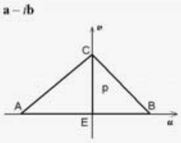



## Algorithm (cont.)

4. Conjugate gradient solver

$$\mathbf{D}(\mathbf{z}^r + i\mathbf{z}^i) = \mathbf{a} - i\mathbf{b}$$

$$\mathbf{a}_Q - i\mathbf{b}_Q = \begin{cases} \frac{-1}{\|B-A\|} + i\frac{1-\theta}{\|C-E\|} & \textit{for } Q = A \\ \frac{1}{\|B-A\|} + i\frac{\theta}{\|C-E\|} & \textit{for } Q = B \\ i\frac{-1}{\|C-E\|} & \textit{for } Q = C \\ 0 & \textit{for others} \end{cases}$$




## Algorithm (cont.)

5. Assign $z^r_P$, $z^i_P$ to each node as their $(x,y)$ coordinates
   ---Map to plane.

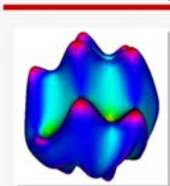
6. Stereographic projection
   ---Map to sphere

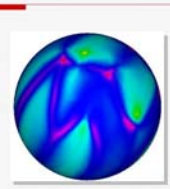
## Algorithm


S. Angenent, S. Haker, A. Tannenbaum, and R. Kikinis, "*On the Laplace-Beltrami operator and brain surface flattening*," IEEE Trans. on Medical Imaging, Vol. 18, pp. 700-711, 1999

Y. Gao, J. Melonakos, A. Tannenbaum. "*Conformal Flattening ITK Filter.*" Insight Journal, 2006. http://hdl.handle.net/1926/225.


## Results and Applications





## Discussion

- As area preserving as possible.
- Used for registration
- Used for remesh the surface